Modelling Clustering as a Result of Autotaxis

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We construct a model describing the process of animal clustering due to autotaxis: the movement of individuals towards their own density gradient. Focusing on projections of midges (*Anarete pritchardii*) movements onto a horizontal plane, Okubo and co-authors [2] indicated that the acceleration of each insect at the centre of the swarm is close to zero and the strongest accelerations towards the centre are at the edge of the swarm. At the swarm centre, midges fly virtually randomly. Similar phenomenon was detected in laboratory observations of juvenile blacksmith (*Chromis punctipinnis*) behaviour [3]. Within a simple Eulerian model, we assume that taxis acceleration is proportional to the gradient of stimulus density. This approach was successfully applied earlier in a model of predator-prey dynamics [1].

Let us describe the population dynamics by the scalar field of density $P = P(\mathbf{x}, t)$ and the vector field of velocity $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$. The model of spatio-temporal dynamics of the swarming group of animals within the closed 2D rectangular habitat $[0, L_1] \times [0, L_2]$ is:

$$\frac{\partial \mathbf{v}}{\partial t} = \kappa \nabla P (1 - P/P_{\max}) + \delta_v \Delta \mathbf{v}; \frac{\partial P}{\partial t} + div(P\mathbf{v}) = \delta_P \Delta P; \qquad (1)$$

where κ, δ_v, δ_P and $P_{\max}$ are parameters. According to (1) and zero-flux

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boundary conditions, the average density of the swarming group P^* does not vary with time.

Linear analysis of (1) on the two-dimensional closed habitat showed that the homogeneous equilibrium loses monotonically its stability with respect to small disturbances corresponding to modes with wave vectors $\mathbf{s}_{nk} = (n\pi/L_1, k\pi/L_2)$ if the following inequality is true: $\kappa P^*(1 - P^*/P_{\max}) > \delta_v \delta_P ((n\pi/L_1)^2 + (k\pi/L_2)^2)$. The obvious necessary condition fulfilling the inequality is $P^* < P_{\max}$. A similar self-organisation mechanism of clustering larvae of a coleopteran (*Dendroctonus micans*) is described in the book [4]: "At high density a cluster appears and rapidly grows at the centre of the experimental setup. At very low density, no stable clusters formed". The results of the analytical investigation were corroborated by numerical simulations with 1D and 2D approximations of model (1). The model demonstrates the occurrence of steady spatial heterogeneous structures interpreted as formation of stationary groups of individuals like shoals or swarms.

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